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An Application of Probability Theory to a Group of Breath-Alcohol and Blood-Alcohol Data

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ABSTRACT: Many jurisdictions have "per se" driving-while-intoxicated (DWI) statutes expressed in terms of a blood-alcohol concentration (BAC) standard (in grams per 100 mL or the equivalent). Since breath-alcohol (BrAC) analysis is typically employed to determine BAC, there is often challenge to the use of an assumed 2100:1 conversion ratio. This concern may be relevant in light of considerable data that show a low percentage of cases in which BrAC > BAC, and this concern increases when the BrAC is used to predict BAC in the context of "per se" legislation.

Probability theory provides a basis for estimating the likelihood of an individual having a BrAC ≥ 0.10 g/210 L with a corresponding BAC < 0.10 g/100 mL. Actual field data from the state of Wisconsin (n = 404) were evaluated to determine the probability of this occurrence. The probability for this occurrence involves the multiplication law for independent events. The computed probability from the data was 0.018. The actual number of occurrences where BrAC ≥ 0.10 g/210 L and BAC < 0.10 g/100 mL was 5, resulting in a probability of 0.012. The concern of having BrAC > BAC at the critical "per se" level has a very low probability of occurrence, which thus supports the reasonableness of "per se" DWI legislation based upon a blood-alcohol standard determined by breath-alcohol analysis.

KEYWORDS: forensic science, alcohol, breath-alcohol testing devices

Many, if not most, jurisdictions in this country have driving-while-intoxicated (DWI) "per se" legislation which define the offence in terms of a specific breath- or blood-alcohol concentration. In most jurisdictions, 0.10 is the critical level of offense and may be stated in terms of either breath-alcohol concentration (BrAC) in grams per 210 L, or blood-alcohol concentration (BAC) in grams per 100 mL. Although there is a trend toward expressing the violation in terms of BrAC, many jurisdictions still have a BAC standard in their statutory language.

The "per se" statutory language makes the 0.10 evidence an irrebuttable presumption and thereby brings increasing focus and scrutiny upon breath-alcohol testing. Jurisdictions combining both "per se" and BAC language face the continual challenge of proving a specific BAC from BrAC evidence. BAC/BrAC ratios and their associated uncertainties are typically the focus of the defense challenge. This is particularly the issue when the defendant's breath-alcohol analysis is at the critical 0.10 level (or other relevant "per se" level).

It is therefore of interest to forensic science to know how many individuals arrested for DWI might be expected to have a BrAC in the region of 0.10 g/210 L and also have

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a BrAC that exceeds its corresponding BAC. The application of basic probability theory provides a means of answering this question. Corresponding BAC and BrAC data can be combined to generate two frequency distributions: the BAC-BrAC difference distribution and the BrAC distribution. Both distributions can be assumed as models for probability distributions and combined to give the probability of an individual having both a particular BrAC and a BAC-BrAC difference which, when combined, would result in an overestimation of BAC by BrAC at the critical "per se" level.

Methods

The data evaluated in the present study were received from the state of Wisconsin and previously published with appropriate regression analyses [*1*]. The data consisted of 404 corresponding blood- and breath-alcohol measurements collected in actual law enforcement situations. The breath-alcohol analyses were performed by trained operators on Breathalyzer® Model 900 and 900A instruments. Blood-alcohol analyses were performed on whole blood specimens by gas chromatography. The blood specimens consisted of venous blood collected no more than one hour after the breath-alcohol analysis (except in nine cases in which the blood was collected prior to the breath). The BAC consisted of one measurement and was reported to three decimal places. The BrAC results were truncated to two decimal places, which is typical in forensic science applications [*2*].

The differences between the BAC and BrAC were computed by

$$\text{DIFF} = \text{BAC} - \text{BrAC}$$

with the results expressed as grams per 100 mL, since the BrAC has an assumed conversion factor of 2100. The resulting differences were rounded to two decimal places, which is consistent with principles of error propagation and uncertainty [*3,4*]. A distribution of differences (DIFF) resulted.

Frequency distributions of both DIFF and BrAC were generated along with relevant parameters. DIFF was then plotted against the mean of the corresponding BAC and BrAC values (BAC + BrAC/2). This method is preferred to either regression or correlation analysis alone and allows confidence intervals to be evaluated [*5*]. The correlation between DIFF and BrAC was also computed and was helpful in determining the degree of independence between the two variables. Correlation rather than regression analysis was used since the data pairs were not selected on the basis of preselected BrAC (independent variable) values [*6*]. It is assumed that the DIFF and BrAC are from a two-variable (bivariate) normal distribution.

Probability estimates were then computed for each situation of concern to forensic science. These situations of concern occur when DIFF is negative and the magnitude is such that a BrAC would exceed the 0.10 level while the BAC would fall below.

Finally, the actual number of times that DIFF was negative and BAC fell below the 0.10 g/100 mL "per se" level was determined. This provided a basis for comparing the probability of occurrence and the actual number of times the event occurred in the data set. All statistical analyses were performed with a program known as SPSS/PC+ on an IBM PC computer.[2]

Results

Figure 1 shows the frequency distribution of DIFF (BAC − BrAC). The distribution spanned from −0.04 to 0.11 g/100 mL, with a mean of 0.018 g/100 mL and a skewness

[2]The software is SPSS/PC +, Version 3.0, manufactured by SPSS, Inc., Chicago, IL.

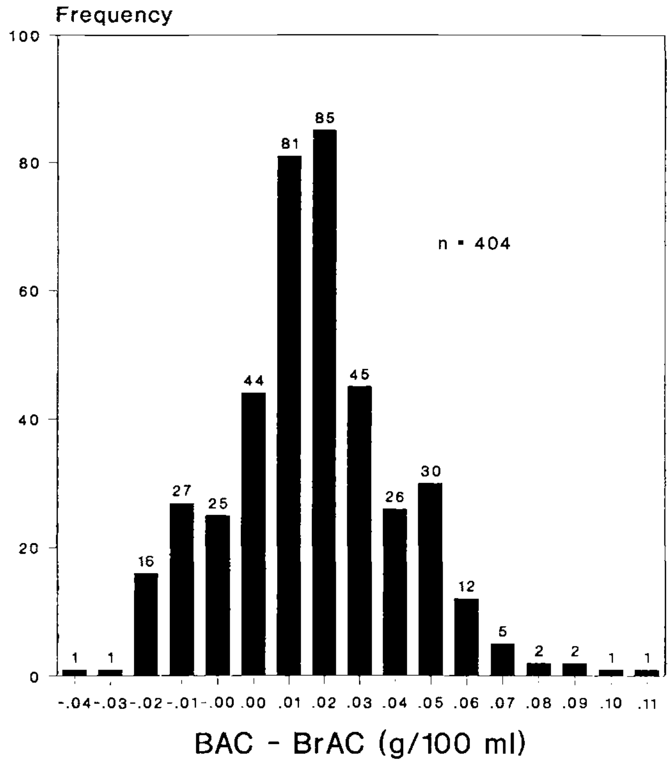

FIG. 1—*Distribution of BAC − BrAC differences (DIFF).*

of 0.749. The interval designated "−0.00" represents cases where the BrAC exceeded the corresponding BAC only in the third decimal place. It is important to note these, however, since the third decimal place is a significant digit. There were 70 negative DIFF values representing cases where the BrAC exceeded the corresponding BAC. These represented 17.3% of the data set. Since the Breathalyzer instruments were calibrated to a BAC/BrAC conversion factor of 2100, the BAC/BrAC ratio K will be less than 2100 in those instances were BrAC > BAC. The ratio K is simply a dimensionless constant computed by dividing BAC by BrAC. It is simply a ratio of two independent measurements.

Figure 2 represents the frequency distribution of BrAC results. Since these are actual field data, the distribution appropriately represents the population of BrAC results from individuals arrested for driving while intoxicated. The BrAC distribution has a mean of 0.157 g/210 L and spanned from 0.00 g/210 L to 0.44 g/210 L.

Figure 3 illustrates two probability distributions approximating the frequency distributions for DIFF and BrAC. These represent limiting distributions which would result as smooth curves if an infinite number of data values were compiled [7]. Figure 3*a* corresponds to the probability distribution for DIFF and shows the area where DIFF $\leq$ -0.02 g/210 L and represents 4.5% of the distribution. Figure 3*b* shows the probability distribution representing the BrAC data, indicates the area falling between 0.10 g/210 L and 0.12 g/210 L, and represents 17.3% of the distribution.

Figure 4 shows the plot of DIFF against the mean of BAC and BrAC. The mean DIFF, along with ± 2 standard deviations (SD), is shown as well. This is similar to a plot of

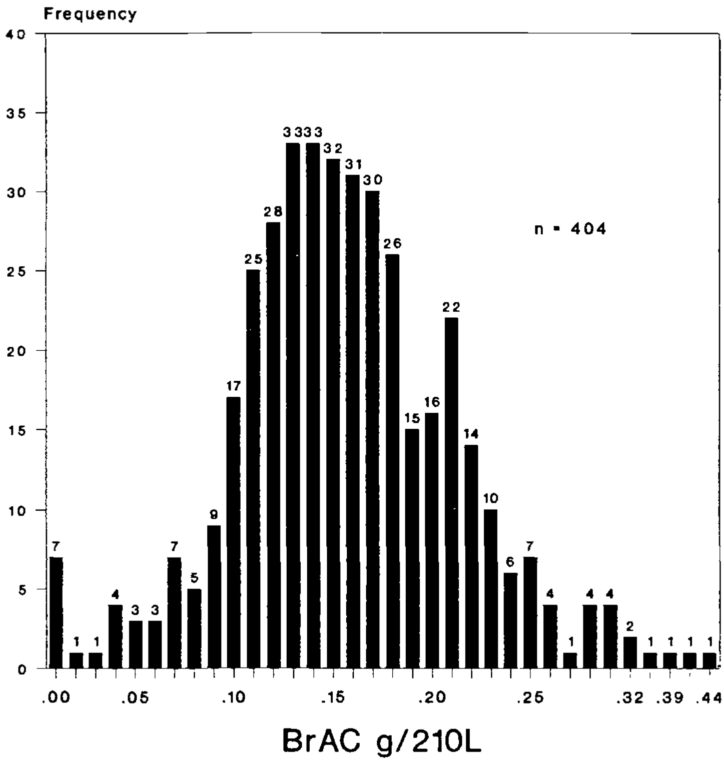

FIG. 2—*Distribution of BrAC results.*

residuals in regression analysis and allows comparison of agreement between two methods that purport to measure the same property throughout their range [*8*]. The correlation coefficient resulted in $r = 0.15$. Figure 4 demonstrates sufficient independence between the two variables (DIFF and mean), even though the differences appear to increase slightly at higher values. The fairly uniform residual variance throughout the measurement range further confirms the independence of the two events and justifies computing the appropriate probability for independent events [*6*]. The probability that any one individual would fall within both areas of each distribution in Fig. 3, given that they are independent events, is simply the product of each individual probability. Their independence means that the occurrence of one event has no affect on the occurrence of the other event. This comes from basic probability theory, stated as [*9–11*]

$$Pr(A \cap B) = Pr(A) \cdot Pr(B)$$

Therefore, based on the present data, the probability that an individual arrested for driving while intoxicated will have both a BrAC between 0.10 g/210 L and 0.12 g/210 L (A) and a corresponding BAC overestimated by 0.02 or greater (B) would be computed as

$$Pr(A \cap B) = (0.173) \cdot (0.045) = 0.0078$$

Table 1 is a summary of the various probability regions of concern in both distributions. The expected probabilities are computed, along with the number of actual occurrences

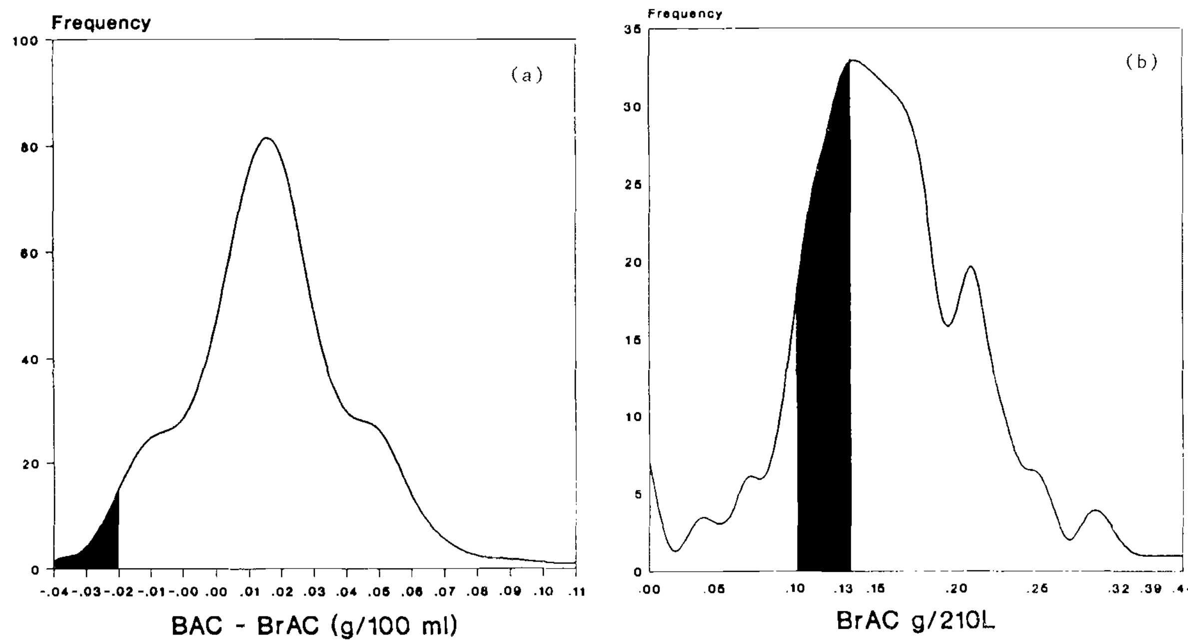

FIG. 3—*Comparing probability regions of both* (a) *the BAC – BrAC distribution and* (b) *the BrAC distribution, smooth approximation.*

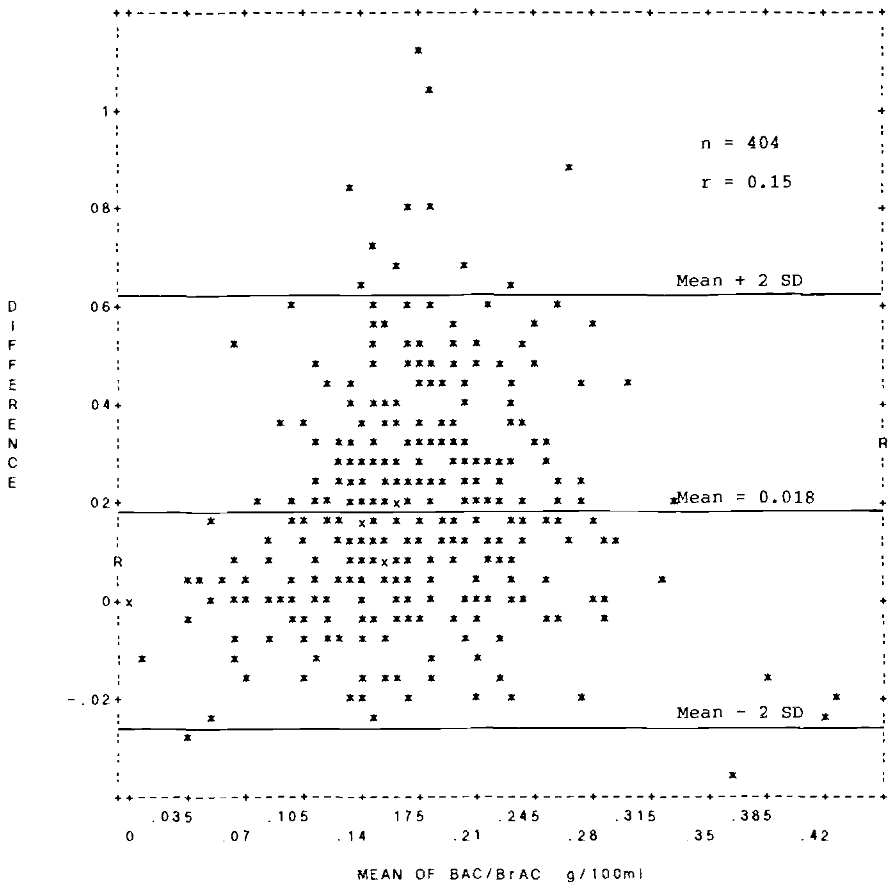

FIG. 4—*Plot of BAC − BrAC difference against mean of BAC and BrAC.*

in the data. The individual probabilities of concern are added together (according to rules for mutually exclusive events) to get a total probability of 0.018 that an individual's BrAC would be greater than 0.10 g/210 L while his or her BAC was less than 0.10 g/100 mL. Table 1 also shows that, in a total of five cases (1.2%) BrAC ≥ 0.10 g/210 L with a corresponding BAC < 0.10 g/100 mL.

Discussion

The adoption of "per se" laws for driving while intoxicated has resulted in increased focus upon breath testing, particularly at the critical level of 0.10 g/210 L. The law makes the level of 0.10 g/210 L a criminal violation, whereas 0.099 g/210 L is not (by a per se standard). It is therefore of interest to know how many individuals may be expected to have breath-alcohol measurements resulting in 0.10 g/210 L or greater with a corresponding BAC < 0.10 g/100 mL.

Jurisdictions with DWI violations in terms of blood-alcohol concentrations (grams per 100 mL or the equivalent) have the difficult task of trying to prove a BAC in terms of BrAC. Many variables exist in reporting a BAC from BrAC analysis where 2100 is the assumed constant, and the issue has been thoroughly reviewed in the literature [*12,13*].

TABLE 1—*Summary of probabilities for various regions of the BAC − BrAC difference distribution and the BrAC distribution.*

BAC − BrAC Occurrence, g/210 L	*Pr*(BAC − BrAC)	Range, g/210 L	*Pr*(BrAC)	*Pr*(BAC − BrAC) · *Pr*(BrAC)	Actual *Pr*(BrAC)
−0.04	0.0025	0.10 to 0.14	0.337	0.0008	0
−0.03	0.0025	0.10 to 0.13	0.255	0.0006	0
−0.02	0.040	0.10 to 0.12	0.173	0.0069	0
−0.01	0.062	0.10 to 0.11	0.104	0.0064	1
−0.001 to −0.004	0.079	0.10	0.042	0.0033	4
				Total 0.018	5 (1.2%)

Several of these variables include the exhalation time, body/breath temperature, type of blood specimen analyzed, state of the absorption/elimination profile, and analytical uncertainty in both methods. Even with the various sources of uncertainty there is a small percentage of samples for which BrAC > BAC in breath-test instruments calibrated to 2100. The present data resulted in 17.4%.

Statistical probability theory provides a basis for quantifying the chances or likelihood of certain events occurring [*14*]. Thus, probability theory provides a basis for comparing the two frequency distributions or events of current interest (DIFF < 0.0 and BrAC > BAC at the per se level). Frequency distributions are the basis for developing probability distributions. even though one describes past observations while the other describes predictive outcome [*15*]. Both frequency distributions seen in Figs. 1 and 2 are developed from the same data set of 404 individuals, thereby enhancing the basis for comparison. Both distributions could be considered two separate events and thus subsets of possible occurrences from a larger population. It is important also that the two events be sufficiently independent. Sufficient independence is verified by the correlation and difference plot seen in Fig. 4.

Similar analysis of a sizable data set from a study in the United Kingdom showed that approximately 2% of the BrAC results exceeded the BAC at the level of 0.08 g/100 mL [*16*]. However, this study utilized time-corrected BAC results for comparison, which may introduce increased uncertainty.

Some researchers have evaluated the BAC/BrAC ratio and attempted to predict the amount of time that BrAC would exceed BAC on that basis [*17*]. Figure 5 shows the correlation between the BAC/BrAC ratio and the mean of BAC and BrAC (BAC + BrAC/2) from the present data. The mean BAC/BrAC ratio along with ±2 SD are shown as well. The BAC/BrAC ratios were computed to two significant figures. It should be noted that the ratio extremes (for example, <1600 and >3000) occur at lower BrAC and BAC values. These ratios should not be assumed, therefore, to apply uniformly throughout all levels of measurement. Other work has noted this concern as well [*18*]. In the BrAC region of 0.10 g/210 L, the ratios are much more consistent. In contrast to those in Fig. 4, the data seen in Fig. 5 are not uniformly distributed throughout the measurement range and are thus not independent events. Probability analysis on the basis of BAC/BrAC ratios would not be justified. A further problem observed in correlating BAC/BrAC ratios to BrAC is the mathematical coupling of data when one variable (BAC/BrAC ratio) is calculated on the basis of the other (BrAC) [*19*]. This can lead to incorrect analysis. Therefore, differences (BAC − BrAC) seem more appropriate to evaluate than BAC/BrAC ratios.

Statistical analysis and probability theory provide the means by which data can be evaluated and conclusions drawn. Virtually nothing in science is concluded without some degree of uncertainty. Science, therefore, relies heavily on probability to determine the likelihood of certain events [*20*]. Statistical probability, therefore, simply provides a tool for rational decision making. This accounts for part of the difficulty in merging science and the law. Scientists use a different language, employing terms such as "significant," "normal," "independent," and "error," all understood in a statistical context [*21*]. On the other hand, the law demands proof beyond a reasonable doubt and seeks to eliminate all uncertainty. It would seem, at least from the present study, that probability theory provides a tool for evaluating the very undesirable situation in which an individual arrested for DWI with 0.10 g/210 L might also have a BAC of less than 0.10 g/100 mL where a "per se" BAC law exists.

Another study also provided a similar approach to this issue on the basis of probability analysis [*22*]. On the basis of field-collected BAC and BrAC data (n = 134), that study computed the probability that a BAC would be equal to or exceed 0.10 g/100 mL for a particular BrAC. With BrAC = 0.10 g/210 L, they computed a probability of 0.95 that

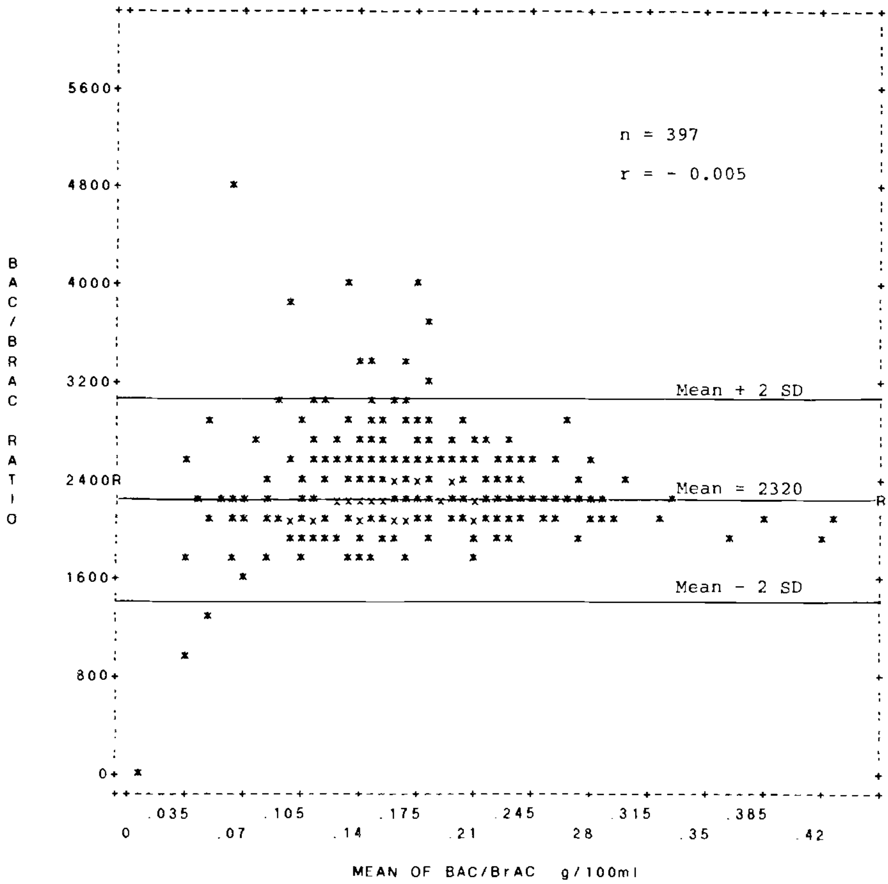

FIG. 5—*Plot of BAC/BrAC ratio against mean of BAC and BrAC.*

BAC ≥ 0.10 g/100 mL. At least the trier of fact should be able to weigh the evidence in light of these types of analyses. Scientific analysis is not intended to prove guilt, but only to provide reliable information for the trier of fact to weigh appropriately.

The present analysis certainly points out the minimal probability of having BrAC > BAC at the critical "per se" level. In light of this minimal probability, "per se" DWI laws based upon a blood-alcohol standard (although measured by breath) are certainly reasonable and with merit. Cases where BrAC > BAC by up to 0.04 g/100 mL are meaningless at BrAC levels other than the critical "per se" level.

Conclusions

Science is frequently called upon to provide evidence in criminal proceedings. This is particularly true in forensic applications of breath-alcohol measurements where tough legislation and penalties exist. Statistics and probability are tools used by scientists to evaluate data and draw reasonable conclusions concerning various observable phenomena.

Historically, breath-alcohol analysis has been used to estimate the corresponding blood-alcohol concentration, which is supported by a large body of data. The forensic application of breath-alcohol results in jurisdictions with both "per se" legislation and a BAC standard results in greater scrutiny of the uncertainties involved. The greatest concern would be if an individual had a BrAC $\geq$ 0.10 g/210 L with a corresponding BAC $<$ 0.10 g/100 mL. Probability theory provides a basis for estimating the likelihood of this event occurring. Analysis of actual field data and the application of appropriate probability methods indicate the probability to be 0.018. This is a very unlikely event and certainly provides insight for justifying the application of DWI "per se" laws based upon blood-alcohol standards.

Acknowledgment

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